**Boyle’s Law: Pressure and Volume**

How would an increase in pressure affect the volume of a contained gas? 

**If the temperature is constant, as the pressure of a gas increases, the volume decreases.** In turn, as the pressure decreases, the volume increases. Robert Boyle was the first person to study this pressure-volume relationship in a systematic way. In 1662, Boyle proposed a law to describe the relationship. **Boyle’s law** states that for a given mass of gas at constant temperature, the volume of the gas varies inversely with pressure.

Look at Figure 14.8. A gas with a volume of 1.0 L ($V_1$) is at a pressure of 100 kPa ($P_1$). As the volume increases to 2.0 L ($V_2$), the pressure decreases to 50 kPa ($P_2$). The product $P_1 \times V_1$ (100 kPa $\times$ 1.0 L = 100 kPa·L) is the same as the product $P_2 \times V_2$ (50 kPa $\times$ 2.0 L = 100 kPa·L). As the volume decreases to 0.5 L ($V_3$), the pressure increases to 200 kPa ($P_3$). Again, the product of the pressure and the volume equals 100 kPa·L.

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**Figure 14.8** The pressure of a gas changes as the volume changes.

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**INTERPRETING GRAPHS**

a. **Observing** When the volume is 2.0 L, what is the pressure?

b. **Predicting** What would the pressure be if the volume were increased to 3.0 L?

c. **Drawing Conclusions** Based on the shape of the graph, describe the general pressure-volume relationship.

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**Enrichment Question**

Pick two points on the graph, and use them to show mathematically that Boyle’s Law describes an inverse relationship. (For this graph, $PV = 100$ kPa·L for any point on the line.)
In an inverse relationship, the product of the two variable quantities is constant. So the product of pressure and volume at any two sets of pressure and volume conditions is always constant at a given temperature. The mathematical expression of Boyle’s law is as follows:

\[ P_1 \times V_1 = P_2 \times V_2 \]

The graph of an inverse relationship is always a curve, as in Figure 14.8.

**SAMPLE PROBLEM 14.1**

**Using Boyle’s Law**

A balloon contains 30.0 L of helium gas at 103 kPa. What is the volume of the helium when the balloon rises to an altitude where the pressure is only 25.0 kPa? (Assume that the temperature remains constant.)

1. **Analyze** List the knowns and the unknown.
   - Knowns: \( P_1 = 103 \text{ kPa} \), \( V_1 = 30.0 \text{ L} \), \( P_2 = 25.0 \text{ kPa} \)
   - Unknown: \( V_2 \)

2. **Calculate** Solve for the unknown.
   - Rearrange Boyle’s law to isolate \( V_2 \):
     \[ V_2 = \frac{V_1 \times P_1}{P_2} \]
   - Substitute the known values for \( P_1 \), \( V_1 \), and \( P_2 \) into the equation and solve.
     \[ V_2 = \frac{30.0 \text{ L} \times 103 \text{ kPa}}{25.0 \text{ kPa}} \]
     \[ = 1.24 \times 10^2 \text{ L} \]

3. **Evaluate** Does the result make sense?
   - A decrease in pressure at constant temperature must correspond to a proportional increase in volume. The calculated result agrees with both kinetic theory and the pressure-volume relationship. Also, the units have canceled correctly and the answer is expressed to the proper number of significant figures.

**Practice Problems**

7. Nitrous oxide (\( N_2O \)) is used as an anesthetic. The pressure on 2.50 L of \( N_2O \) changes from 105 kPa to 40.5 kPa. If the temperature does not change, what will the new volume be?

8. A gas with a volume of 4.00 L at a pressure of 205 kPa is allowed to expand to a volume of 12.0 L. What is the pressure in the container if the temperature remains constant?

**Differentiated Instruction**

**Special Needs**

Consider having students work in pairs to solve the practice problems in this chapter. Match up students who have a mastery of algebraic equations with students who need more practice solving for an unknown.

**Discuss**

Have students consider what will happen when a helium-filled balloon is released into the sky. Assume the temperature remains constant. Remind students that, as the elevation increases, the atmospheric pressure decreases. Ask, If the balloon contains 30 L of gas at 100 kPa, what would its volume be at 25 kPa? (120 L)

**Pressure and Volume**

**Purpose** Students observe the effect that changing pressure has on the volume of a gas.

**Materials** vacuum pump, bell jar, marshmallows

**Procedure** Explain that marshmallows contain trapped air. Place several marshmallows in the bell jar, and then pull a vacuum in the jar. Ask students to explain why the marshmallows increase in size.

**Expected Outcome** The removal of air surrounding the marshmallows reduces the pressure on them. Air trapped inside the marshmallows can expand to a greater volume.

**FYI**

See Explosives on R26 of the Elements Handbook for an example of what can happen when a confined gas under pressure is allowed to expand.
Section 14.2 (continued)

Charles’s Law:
Temperature and Volume

Discuss
Point out that when the pressure and amount of a gas are unchanged, the ratio of the volume of the gas to the absolute temperature of the gas is a constant. Mathematically, this constant can be expressed as \( V_1/T_1 = V_2/T_2 \) and is known as Charles’s law.

Interpreting Graphs

a. kelvin
b. The volume increases
c. The volume would be 0.

Enrichment Question
Point out the \( \Delta V \) and \( \Delta T \) labels. Ask, What does the ratio \( \Delta V/\Delta T \) represent? (slope)

Relate
Jacques Charles was one of three passengers in the second balloon ascension that carried humans. The hydrogen-filled balloon was launched on December 1, 1783, in Paris, France. More than 400,000 curious onlookers attended. Have interested students research the history of ballooning. Have them prepare a report for the class, which includes the types of gases used and the challenges involved.

Charles’s Law: Temperature and Volume

Figure 14.9 When the gas in the blue balloon is cooled at constant pressure, the volume of the gas decreases. When the gas is heated at constant pressure, the volume increases.

Calculating What is the ratio of volume to temperature for each set of conditions? Round your answer to two significant figures.

Figure 14.10 This graph shows how the volume changes as the temperature of a gas changes.

INTERPRETING GRAPHS

a. Observing What is the unit of temperature?
b. Drawing Conclusions What happens to the volume as the temperature rises?
c. Predicting If the temperature of a gas were 0 K, what would the volume of the gas be?

Charles’s Law states that the volume of a fixed mass of gas is directly proportional to its Kelvin temperature if the pressure is kept constant. As the temperature of an enclosed gas increases, the volume increases, if the pressure is constant.

In 1787, the French physicist Jacques Charles studied the effect of temperature on the volume of a gas at constant pressure. When he graphed his data, Charles observed that a graph of gas volume versus temperature (in °C) is a straight line for any gas. When he extrapolated, or extended, the line to zero volume (\( V = 0 \)), the line always intersected the temperature axis at \(-273.15^\circ C\). This value is equal to 0 on the Kelvin temperature scale.

The observations that Charles made are summarized in Charles’s law. Charles’s law states that the volume of a fixed mass of gas is directly proportional to its Kelvin temperature if the pressure is kept constant. Look at the graph in Figure 14.10. When the temperature is 300 K, the volume is 1.0 L. When the temperature is 900 K, the volume is 3.0 L. In both cases, the ratio of \( V \) to \( T \) is 0.0033.

Differentiated Instruction

Gifted and Talented
Explain to students that Charles summarized his observations of the relationship between the volume and temperature of a gas in the following equation: \( V = V_0(1 + aT) \) where \( V_0 \) is the volume of the gas at 0°C, \( T \) is its temperature expressed in °C, and \( a \) is a constant for all gases. Have students show that the numerical value of \( a \) is approximately 1/273.
The ratio \( V_1 / T_1 \) is equal to the ratio \( V_2 / T_2 \). Because this ratio is constant at all conditions of temperature and volume, when the pressure is constant, you can write Charles’s law as follows.

\[
\frac{V_1}{T_1} = \frac{V_2}{T_2}
\]

The ratio of the variables is always a constant in a direct relationship, and the graph is always a straight line. It is not a direct relationship if the temperatures are expressed in degrees Celsius. So when you solve gas law problems, the temperature must always be expressed in kelvins.

### SAMPLE PROBLEM 14.2

**Using Charles’s Law**

A balloon inflated in a room at 24°C has a volume of 4.00 L. The balloon is then heated to a temperature of 58°C. What is the new volume if the pressure remains constant?

1. **Analyze** List the knowns and the unknown.

   **Knowns**
   - \( V_1 = 4.00 \text{ L} \)
   - \( T_1 = 24^\circ \text{C} \)
   - \( T_2 = 58^\circ \text{C} \)

   **Unknown**
   - \( V_2 = ? \text{ L} \)

   Use Charles’s law \( (V_1/T_1) = (V_2/T_2) \) to calculate the unknown value \( V_2 \).

2. **Calculate** Solve for the unknown.

   Because you will use a gas law, express the temperatures in kelvins.

   \[
   T_1 = 24^\circ \text{C} + 273 = 297 \text{ K}
   \]

   \[
   T_2 = 58^\circ \text{C} + 273 = 331 \text{ K}
   \]

   Rearrange Charles’s law to isolate \( V_2 \).

   \[
   V_2 = \frac{V_1 \times T_2}{T_1}
   \]

   Substitute the known values for \( T_1 \), \( V_1 \), and \( T_2 \) into the equation and solve.

   \[
   V_2 = \frac{4.00 \text{ L} \times 331 \text{ K}}{297 \text{ K}} = 4.46 \text{ L}
   \]

3. **Evaluate** Does the result make sense?

   The volume increases as the temperature increases. This result agrees with both the kinetic theory and Charles’s law.

### Practice Problems

**9.** If a sample of gas occupies 6.80 L at 325°C, what will its volume be at 25°C if the pressure does not change?

**10.** Exactly 5.00 L of air at −50.0°C is warmed to 100.0°C. What is the new volume if the pressure remains constant?
Gay-Lussac’s Law: Pressure and Temperature

Relate

Use tire pressure to discuss the relationship between gas pressure and Kelvin temperature. Ask, Why do auto tire manufacturers recommend checking for proper inflation before driving the car more than a mile? (The tires get warm as the car moves, increasing the pressure inside the tires.)

CLASS Activity

Observing the Effect of Pressure on Temperature

Purpose Students observe the relationship between the pressure and temperature of a gas.

Materials 2 large vats, ice water, hot water, inflated bicycle tire

Procedure Fill a large vat with ice water and a second vat with hot water. First, have students squeeze an inflated bicycle tire to assess its firmness. Next, immerse the tire in ice water and have them feel its firmness. Finally, immerse the tire in hot water and assess the firmness. Have the students describe the relationship between pressure and temperature at constant volume.

Expected Outcome A direct relationship exists between pressure and temperature at constant volume.

Gay-Lussac’s Law: Pressure and Temperature

When tires are not inflated to the recommended pressure, fuel efficiency and traction decrease. Treads can wear down faster. Most importantly, improper inflation can lead to tire failure. A driver should not check tire pressure after driving a long distance because the air in a tire heats up during a drive. As the temperature of an enclosed gas increases, the pressure increases, if the volume is constant.

Joseph Gay-Lussac (1778–1850), a French chemist, discovered the relationship between the pressure and temperature of a gas in 1802. His name is on the gas law that describes the relationship. Gay-Lussac’s law states that the pressure of a gas is directly proportional to the Kelvin temperature if the volume remains constant. Look at Figure 14.11. When the temperature is 300 K, the pressure is 100 kPa. When the temperature is doubled to 600 K, the pressure doubles to 200 kPa. Because Gay-Lussac’s law involves direct proportions, the ratios $P_1/T_1$ and $P_2/T_2$ are equal at constant volume. You can write Gay-Lussac’s law as follows.

\[
\frac{P_1}{T_1} = \frac{P_2}{T_2}
\]

Gay-Lussac’s law can be applied to reduce the time it takes to cook food. One cooking method involves placing food above a layer of water and heating the water. The water vapor, or steam, that is produced cooks the food. Steam that escapes from the pot is at a temperature of about 100°C when the pressure is near one atmosphere. In a pressure cooker, like the one shown in Figure 14.12, steam is trapped inside the cooker. The temperature of the steam reaches about 120°C. The food cooks faster at this higher temperature, but the pressure rises, which increases the risk of an explosion. A pressure cooker has a valve that allows some vapor to escape when the pressure exceeds the set value.

Checkpoint How does a pressure cooker affect the time needed to cook food?
SAMPLE PROBLEM 14.3

Using Gay-Lussac’s Law

Aerosol cans carry warnings on their labels that say not to incinerate (burn) them or store the cans above a certain temperature. This problem will show why it is dangerous to dispose of aerosol cans in a fire. If the can is thrown onto a fire, what will the pressure be when the temperature reaches 928°C?

1. Analyze  List the knowns and the unknown.
   - Knowns
     • \( P_1 = 103 \text{ kPa} \)
     • \( T_1 = 25°C \)
     • \( T_2 = 928°C \)
   - Unknown
     • \( P_2 \)

   Use Gay-Lussac’s law \( (P_1/T_1 = P_2/T_2) \) to calculate the unknown \( P_2 \). Remember, because this problem involves temperatures and a gas law, the temperatures must be expressed in kelvins.

2. Calculate  Solve for the unknown.
   - First convert degrees Celsius to kelvins.
     \[
     T_1 = 25°C + 273 = 298 \text{ K} \\
     T_2 = 928°C + 273 = 1201 \text{ K}
     \]
   - Rearrange Gay-Lussac’s law to isolate \( P_2 \).
     \[
     P_2 = \frac{P_1 \times T_2}{T_1}
     \]
   - Substitute the known values for \( P_1, T_2, \) and \( T_1 \) into the equation and solve.
     \[
     P_2 = \frac{103 \text{ kPa} \times 1201 \text{ K}}{298 \text{ K}} = 415 \text{ kPa} = 4.15 \times 10^2 \text{ kPa}
     \]

3. Evaluate  Does the result make sense?
   - From the kinetic theory, one would expect the increase in temperature of a gas to produce an increase in pressure if the volume remains constant. The calculated value does show such an increase.

Practice Problems

11. A sample of nitrogen gas has a pressure of 6.58 kPa at 539 K. If the volume does not change, what will the pressure be at 211 K?

12. The pressure in a car tire is 198 kPa at 27°C. After a long drive, the pressure is 225 kPa. What is the temperature of the air in the tire? Assume that the volume is constant.

Differentiated Instruction

Gifted and Talented

Gases such as oxygen, nitrogen, helium, and hydrogen can be liquefied by lowering the temperature and increasing the pressure. But different gases must be cooled to different temperatures before they condense, no matter how high the pressure is raised. This temperature is called the “critical temperature.” Have students look up the critical temperatures for oxygen, nitrogen, helium, and hydrogen. Ask, What do these temperatures indicate about the relative strengths of the intermolecular attractions for these gases? (The lower the critical temperature, the lower the intermolecular attractive forces.)

Answers

11. \( 6.58 \text{ kPa}/539 \text{ K} = P_2/211 \text{ K} \)
   \[
   P_2 = 211 \text{ K} \times 6.58 \text{ kPa}/539 \text{ K} = 2.58 \text{ kPa}
   \]

12. \( T_1 = 27°C + 273 = 300 \text{ K} \)
   \[
   198 \text{ kPa}/300 \text{ K} = 225 \text{ kPa}/T_2 \\
   T_2 = 225 \text{ kPa} \times 300 \text{ K}/198 \text{ kPa} = 341 \text{ K} (68°C)
   \]

Practice Problems Plus

The pressure in a sealed plastic container is 108 kPa at 41°C. What is the pressure when the temperature drops to 22°C? Assume that the volume has not changed.

Discuss

The direct relationship between pressure and temperature at constant volume can be expressed as \( P_1/T_1 = P_2/T_2 \). In Gay-Lussac’s law problems, the new pressure of the gas is equal to its original pressure times a quotient. The value of the quotient indicates whether the gas is heated or cooled. If the gas is heated \( (T_2 > T_1) \), the new pressure is greater. So, the quotient has to be greater than one \( (T_2/T_1) \). If the gas is cooled \( (T_2 < T_1) \), the new pressure is less. So, the quotient must be less than one \( (T_1/T_2) \).

Answers to...

Figure 14.11 The number of particles in each container is the same.

A pressure cooker reduces cooking time.
The Combined Gas Law

Sample Problem 14.4

Answers

13. \( T_1 = 25°C + 273 = 298 \text{ K} \)
   \( T_2 = 125°C + 273 = 398 \text{ K} \)
   155 kPa \( \times \) 1.00 L/298 K =
   605 kPa \( \times \) \( V_2/398 \text{ K} \)
   \( V_2 = 398 \text{ K} \times 155 \text{ kPa} \times 1.00 \text{ L}/298 \text{ K} \times 605 \text{ kPa} = 0.342 \text{ L} \)
14. \( T_1 = -50°C + 273 = 223 \text{ K} \)
   \( T_2 = 102°C + 273 = 375 \text{ K} \)
   107 kPa \( \times \) 5.00 L/223 K =
   \( P_2 \times 7.00 \text{ L}/375 \text{ K} \)
   \( P_2 = 375 \text{ K} \times 107 \text{ kPa} \times 5.00 \text{ L}/223 \text{ K} \times 7.00 \text{ L} = 1.29 \times 10^4 \text{ kPa} \)

Practice Problems Plus

The volume of a gas at 26°C and 75 kPa is 10.5 L. The pressure is increased to 116 kPa. What final temperature would be required to reduce the volume to 9.5 L? (418 K or 145°C)

Discuss

Write the equation for the combined gas law on the board. Ask, What variable that is used to describe a gas is missing from this equation? (\( n \), the number of moles of gas)

The ascent of a weather balloon is representative of many events that occur outside a laboratory. Instead of a controlled experiment with manipulated and responding variables (with all other variables controlled), multiple variables are changing simultaneously.

Differentiated Instruction

Gifted and Talented

Have students use the combined gas law to derive the equations for Charles's law and Gay-Lussac's law. (The variables \( n \) and \( P \) are constant for Charles's law, and \( n \) and \( V \) are constant for Gay-Lussac's law.)
Weather balloons, like the one in Figure 14.13, carry a package of data-gathering instruments up into the atmosphere. At an altitude of about 27,000 meters, the balloon bursts. The combined gas law can help to explain this situation. Both outside temperature and pressure drop as the balloon rises. These changes have opposite effects on the volume of the weather balloon. A drop in temperature causes the volume of an enclosed gas to decrease. A drop in outside pressure causes the volume to increase. Given that the balloon bursts, the drop in pressure must affect the volume more than the drop in temperature does.

The combined gas law can also help you solve gas problems when only two variables are changing. It may seem challenging to remember four different expressions for the gas laws. But you actually only need to remember two laws from the combined gas law by holding one variable constant.

To illustrate, suppose you hold the temperature constant ($T_1 = T_2$). Rearrange the combined gas law so that the two temperature terms are on the same side of the equation. Because $T_1 = T_2$, the ratio of $T_1$ to $T_2$ is equal to one. Multiplying by 1 does not change a value in an equation. So when the temperature is constant, you can delete the temperature ratio from the rearranged combined gas law. What you are left with is the mathematical expression of Boyle’s law.

$$P_1 \times V_1 = P_2 \times V_2 \times \frac{T_1}{T_2}$$

A similar process yields Charles’s law when pressure remains constant and Gay-Lussac’s law when volume remains constant.

## 14.2 Section Assessment

15. **Key Concept** How are the pressure and volume of a gas related at constant temperature?

16. **Key Concept** If pressure is constant, how does a change in temperature affect the volume of a gas?

17. **Key Concept** What is the relationship between the temperature and pressure of a contained gas at constant volume?

18. **Key Concept** In what situations is the combined gas law useful?

19. Write the mathematical equation for Boyle’s law and explain the symbols.

20. A given mass of air has a volume of 6.00 L at 101 kPa. What volume will it occupy at 25.0 kPa if the temperature does not change?

21. Explain how Charles’s law can be derived from the combined gas law.

22. The volume of a weather balloon increases as the balloon rises in the atmosphere. Why doesn’t the drop in temperature at higher altitudes cause the volume to decrease?

## Connecting Concepts

### Phase Changes

Use what you learned about phase changes in Section 13.4 to try to explain this statement: Scientists cannot collect temperature and volume data for an enclosed gas at temperatures near absolute zero.

### Predicting

Explain why helium is more likely than air to be used in weather balloons.

### Reteach

In many situations, a sample of gas is simultaneously subjected to pressure and temperature changes that have opposite effects on volume. Consider a weather balloon rising through the atmosphere. The higher it rises, the colder the temperature and the lower the volume. At the same time, atmospheric pressure decreases, allowing the gas to expand. The combined gas law allows students to determine which variable has the greater effect on the volume.

## Connecting Concepts

As temperatures decrease toward absolute zero, particles in the gas slow down, and attractions between particles increase. A gas would liquefy, then solidify, as it is cooled to temperatures near absolute zero.