### FOCUS

**Objectives**

1. **16.4.1 Solve problems** related to the molality and mole fraction of a solution

2. **16.4.2 Describe** how freezing-point depression and boiling-point elevation are related to molality.

### Guide to Reading

**Build Vocabulary**

Use a chart to organize the definitions and the mathematical formulas associated with each vocabulary term.

**Reading Strategy**

Have students preview this section by reading the key concepts and skimming the headings, visuals, and boldfaced materials.

### INSTRUCT

**Connecting to Your World**

Cooking instructions for a wide variety of foods, from dried pasta to packaged beans to frozen fruits to fresh vegetables, often call for the addition of a small amount of salt to the cooking water. Most people like the flavor of food cooked with salt. But adding salt can have another effect on the cooking process. Recall that dissolved salt elevates the boiling point of water. Suppose you added a teaspoon of salt to two liters of water. A teaspoon of salt has a mass of about 20 g. Would the resulting boiling point increase be enough to shorten the time required for cooking? In this section, you will learn how to calculate the amount the boiling point of the cooking water would rise.

### Molality and Mole Fraction

Recall that colligative properties depend only upon solute concentration.

The **unit molality and mole fractions** are two additional ways in which chemists express the concentration of a solution. The unit **molality** \( m \) is the number of moles of solute dissolved in 1 kilogram (1000 g) of solvent. Molality is also known as molal concentration.

\[
\text{Molality} = \frac{\text{moles of solute}}{\text{kilogram of solvent}}
\]

Note that molality is not the same as molarity. Molality refers to moles of solute per kilogram of solvent rather than moles of solute per liter of solution. In the case of water as the solvent, 1 kg or 1000 g equals a volume of 1000 mL, or 1 L.

You can prepare a solution that is 1.00 molal (1 \( m \)) in glucose, for example, by adding 1.00 mol (180 g) of glucose to 1000 g of water. Figure 16.17 shows how a 0.500 molal \( (0.500 \ m) \) solution in sodium chloride is prepared by dissolving 0.500 mol (29.3 g) of NaCl in 1.000 kg (1000 g) of water.

**Guide for Reading**

**Key Concepts**

- What are two ways of expressing the concentration of a solution?
- How are freezing-point depression and boiling-point elevation related to molality?

**Vocabulary**

- molality \( (m) \)
- mole fraction
- molal freezing-point depression constant \( (K_f) \)
- molal boiling-point elevation constant \( (K_b) \)

**Reading Strategy**

Before you read, make a list of the vocabulary terms above. As you read, write the symbols or formulas that apply to each term and describe them using words.

**INSTRUCT**

Ask, **What effect does adding salt have on the cooking process?** (Adding salt increases the boiling point of the cooking water.) Ask, **Do you think the resulting boiling point increase would be enough to significantly shorten the time required for cooking?** (Students are likely to predict, correctly, that the change would be negligible.)

### Molality and Mole Fraction

**Use Visuals**

Figure 16.17 Display the figure on an overhead projector. Ask students to write the definition of molality in their notebooks. Show the step-by-step procedure a chemist would use to prepare a 0.500\( m \) solution of NaCl. Have students confirm your calculations and the data given in the figure.

**Answers to...**

**Figure 16.17 1.00m**
Section 16.4 (continued)

Sample Problem 16.6

Answers

29. 750 g water × 0.400 mol NaF / 1000 g water × 42.0 g NaF / 1 mol NaF = 1.26 × 10⁻¹ g NaF

30. 10.0 g NaCl / 600 g water × 1 mol NaCl / 58.5 g NaCl × 1000 g water / 1 kg water = 2.85 × 10⁻¹ m NaCl

Practice Problems Plus

How many grams of lithium bromide must be dissolved in 444 g of water to prepare a 0.140 m LiBr solution? (5.40 g LiBr)

Discuss

Write the expressions defining molarity and molality on the board. Compare the chemical quantities in each expression. Point out that molarity is denoted by M and molality by m. Explain that the molality of a solution does not vary with temperature because the mass of the solvent does not change. In contrast, the molarity of a solution does vary with temperature because the liquid can expand and contract. When studying colligative properties such as boiling-point elevation and freezing-point depression, it is preferable to use a concentration that does not depend on temperature.

SAMPLE PROBLEM 16.6

Using Solution Molality

How many grams of potassium iodide must be dissolved in 500.0 g of water to produce a 0.060 molal KI solution?

1. Analyze List the knowns and the unknown.

   Knowns
   - mass of water = 500.0 g = 0.5000 kg
   - solution concentration = 0.060 m
   - molar mass KI = 166.0 g/mol

   Unknown
   - mass of solute = ? g KI

   According to the definition of molal, the final solution must contain 0.060 mol KI per 1000 g H₂O. Use the molality as a conversion factor to convert from mass of water to moles of the solute (KI). Then use the molar mass of KI to convert from mol KI to g KI. The steps are:

   mass of H₂O → mol KI → g KI.

2. Calculate Solve for the unknown.

   0.5000 kg H₂O × 0.060 mol KI / 1000 kg H₂O × 166.0 g KI / 1 mol KI = 5.0 g KI

3. Evaluate Does the result make sense?

A 1 molal KI solution is one molar mass of KI (166.0 g) dissolved in 1000 g of water. The desired molal concentration (0.060 m) is about 1/20 of that value, so the mass of KI should be much less than the molar mass. The answer is correctly expressed to two significant figures.

Math Handbook

For help with dimensional analysis, go to page R66.

Interactive Textbook

Problem-Solving 16.29 Solve Problem 29 with the help of an interactive guided tutorial. with ChemASAP

Practice Problems

29. How many grams of sodium fluoride are needed to prepare a 0.400 m NaF solution that contains 750 g of water?

30. Calculate the molality of a solution prepared by dissolving 10.0 g NaCl in 600 g of water.

The concentration of a solution can also be expressed as a mole fraction, as shown in Figure 16.18. The mole fraction of a solute in a solution is the ratio of the moles of that solute to the total number of moles of solvent and solute. In a solution containing n₁ mol of solute A and n₂ mol of solvent B, the mole fraction of solute A (Xₐ) and the mole fraction of solvent B (Xₜ) can be expressed as:

\[ X_a = \frac{n_a}{n_a + n_b} \]
\[ X_t = \frac{n_b}{n_a + n_b} \]

Differentiated Instruction

Less Proficient Readers

Have students write sentences using the words molarity and molality, and have them circle the letter in each word that makes them distinct. The mnemonic, ‘r’ for molarity and liter may help.

Facts and Figures

Mole Fraction Uses

Explain that the mole fraction compares the number of moles of a solute to the total number of moles in the solution. Organic chemists, who frequently work with non-aqueous solvent systems, often use this method of expressing concentration. The mole fraction is also used when calculating the vapor pressure of a solution.
SAMPLE PROBLEM 16.7

Calculating Mole Fractions

Ethylene glycol (C\(_2\)H\(_4\)O\(_2\)) is added to automobile cooling systems to protect against cold weather. What is the mole fraction of each component in a solution containing 1.25 mol of ethylene glycol (EG) and 4.00 mol of water?

1. **Analyze** List the knowns and the unknowns.

   **Knowns**
   - moles of ethylene glycol (\(n_{EG}\)) = 1.25 mol EG
   - moles of water (\(n_{H2O}\)) = 4.00 mol H\(_2\)O

   **Unknowns**
   - mole fraction EG (\(X_{EG}\)) = ?
   - mole fraction H\(_2\)O (\(X_{H2O}\)) = ?

   The mole fraction of ethylene glycol (\(X_{EG}\)) in the solution is the number of moles of ethylene glycol divided by the total number of moles in the solution:

   \[
   X_{EG} = \frac{n_{EG}}{n_{EG} + n_{H2O}}
   \]

   Similarly, the mole fraction of water (\(X_{H2O}\)) in the solution is the number of moles of water divided by the total number of moles in the solution:

   \[
   X_{H2O} = \frac{n_{H2O}}{n_{EG} + n_{H2O}}
   \]

2. **Calculate** Solve for the unknowns.

   \[
   X_{EG} = \frac{1.25 \text{ mol}}{1.25 \text{ mol} + 4.00 \text{ mol}} = 0.238
   \]

   \[
   X_{H2O} = \frac{4.00 \text{ mol}}{1.25 \text{ mol} + 4.00 \text{ mol}} = 0.762
   \]

3. **Evaluate** Does the result make sense?

   The mole fraction is a dimensionless quantity. The sum of the mole fractions of all the components in a solution must equal 1. Note that \(X_{EG} + X_{H2O} = 1.000\). Each answer is correctly expressed to three significant figures.

**Practice Problems**

31. What is the mole fraction of each component in a solution made by mixing 300 g of ethanol (C\(_2\)H\(_5\)OH) and 500 g of water?

32. A solution contains 50.0 g of carbon tetrachloride (CCl\(_4\)) and 50.0 g of chloroform (CHCl\(_3\)). Calculate the mole fraction of each component in the solution.
Freezing-Point Depression and Boiling-Point Elevation

Interpreting Graphs

a. 0°C; 100°C

b. The freezing point of the solution is lower than that of pure water and the boiling point is higher than that of pure water.

c. Adding a solute to water allows it to remain as a liquid over a longer temperature range because the solution changes to a solid at a lower temperature and changes to a vapor at a higher temperature.

Enrichment Question

Which aqueous solution would have the larger boiling point elevation and freezing point depression: 1 m KCl or 1 m CaBr₂? (CaBr₂)

Use Visuals

Figure 16.20 Display Figure 16.20 on an overhead projector. Choose an arbitrary concentration for an aqueous solution of NaCl or ethylene glycol. Calculate the boiling-point elevation and freezing-point depression. Then write the temperatures on the horizontal axis. Show students how to read the data for the boiling points of the pure solvent and of the solution. Then have students read the corresponding values for the freezing points. Ask, What are \( \Delta T_b \) and \( \Delta T_f \) for this solution? (Point out that the solute affects both the freezing point and boiling point of a liquid.) Why would knowing the boiling-point elevation and freezing-point depression be important when choosing antifreeze for car radiators? (Despite its name, antifreeze protects against both freezing and overheating.)

Freezing-Point Depression and Boiling-Point Elevation

The graph in Figure 16.20 shows that the freezing point of a solvent is lowered and its boiling point is raised by the addition of a nonvolatile solute. The magnitudes of the freezing-point depression (\( \Delta T_f \)) and the boiling-point elevation (\( \Delta T_b \)) of a solution are directly proportional to the molal concentration (\( m \)), when the solute is molecular, not ionic.

\[
\Delta T_f = K_f \times m \\
\Delta T_b = K_b \times m
\]

The change in the freezing temperature (\( \Delta T_f \)) is the difference between the freezing point of the solution and the freezing point of the pure solvent. Similarly, the change in the boiling temperature (\( \Delta T_b \)) is the difference between the boiling point of the solution and the boiling point of the pure solvent. The term \( m \) is the molal concentration of the solution.

With the addition of a constant, the proportionality between the freezing point depression (\( \Delta T_f \)) and the molality \( m \) can be expressed as an equation.

\[
\Delta T_f = K_f \times m
\]

The constant, \( K_f \), is the **molal freezing-point depression constant**, which is equal to the change in freezing point for a 1-molal solution of a nonvolatile molecular solute. The value of \( K_f \) depends upon the solvent. Its units are °C/m. Table 16.2 lists the \( K_f \) values for water and some other solvents.

The boiling-point elevation of a solution can also be expressed as an equation.

\[
\Delta T_b = K_b \times m
\]

The constant, \( K_b \), is the **molal boiling-point elevation constant**, which is equal to the change in boiling point for a 1-molal solution of a nonvolatile molecular solute. The value of \( K_b \) depends upon the solvent. Its units are °C/m. Table 16.3 lists the \( K_b \) values for water and some other solvents.

<table>
<thead>
<tr>
<th>Table 16.2</th>
<th>( K_f ) Values for Some Common Solvents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solvent</td>
<td>( K_f ) (°C/m)</td>
</tr>
<tr>
<td>Water</td>
<td>1.86</td>
</tr>
<tr>
<td>Acetic acid</td>
<td>3.90</td>
</tr>
<tr>
<td>Benzene</td>
<td>5.12</td>
</tr>
<tr>
<td>Nitrobenzene</td>
<td>7.00</td>
</tr>
<tr>
<td>Phenol</td>
<td>7.40</td>
</tr>
<tr>
<td>Cyclohexane</td>
<td>20.2</td>
</tr>
<tr>
<td>Camphor</td>
<td>37.7</td>
</tr>
</tbody>
</table>

**Simulation 21** Discover the principle underlying the colligative properties of solutions. With ChemASAP

**Interactive Textbook**

**INTERPRETING GRAPHS**

a. **Identify** What is the freezing point of water? What is the boiling point? b. **Describe** How do the freezing and boiling points of the solution compare to those of pure water? c. **Apply Concepts** Does adding a solute to water allow it to remain as a liquid over a longer or shorter temperature range? Explain.
Sample Problem 16.8 shows how these equations can be used for calculating $\Delta T_f$ and $\Delta T_b$ of solutions if the solute is a molecular compound. For ionic compounds, both the freezing-point depression and the boiling-point elevation depend upon the number of ions produced by each formula unit. This number is used to calculate an effective molality, as shown in Sample Problem 16.9.

**SAMPLE PROBLEM 16.8**

Calculating the Freezing-Point Depression of a Solution

Antifreeze protects a car from freezing. It also protects it from overheating. Calculate the freezing-point depression and the freezing point of a solution containing 100 g of ethylene glycol (C$_2$H$_6$O$_2$) antifreeze in 0.500 kg of water.

1. **Analyze** List the knowns and the unknown.

   **Knowns**
   - mass of solute = 100 g C$_2$H$_6$O$_2$
   - mass of solution = 0.500 kg H$_2$O
   - $K_f$ for H$_2$O = 1.86°C/m
   - $\Delta T_f = K_f \times m$

2. **Calculate** Solve for the unknown.

   moles C$_2$H$_6$O$_2$ = 100 g C$_2$H$_6$O$_2$ / 62.0 g C$_2$H$_6$O$_2$ = 1.61 mol

   $m = \frac{\text{mol solute}}{\text{kg solvent}} = \frac{1.61 \text{ mol}}{0.500 \text{ kg}} = 3.22 m$

   $\Delta T_f = K_f \times m = 1.86^\circ \text{C/m} \times 3.22 m = 5.99^\circ \text{C}$

   The freezing point of the solution is 0.00°C - 5.99°C = -5.99°C.

3. **Evaluate** Does the result make sense?

   A 1-mol solution reduces the freezing temperature by 1.86°C, so a decrease of 5.99°C for an approximately 3-molar solution is reasonable. The answer is correctly expressed with three significant figures.

**Practice Problems**

33. What is the freezing point depression of an aqueous solution of 10.0 g of glucose (C$_6$H$_12$O$_6$) in 50.0 g H$_2$O?

34. Calculate the freezing-point depression of a benzene solution containing 400 g of benzene and 200 g of the molecular compound acetone (C$_3$H$_6$O). $K_f$ for benzene is 5.12°C/m.

---

**Answers**

33. mol C$_6$H$_12$O$_6$ = 10.0 g glucose × 1 mol / 180.2 g = 0.0555 mol; $m =$ 0.0555 mol / 50.0 g H$_2$O × 1000 g / 1 kg = 1.11 m; $\Delta T_f = K_f \times m =$ 1.86°C/m × 1.11 m = 2.06°C

34. mol C$_3$H$_6$O = 200 g C$_3$H$_6$O × 1 mol / 58.0 g C$_3$H$_6$O = 3.45 mol C$_3$H$_6$O; $m =$ 3.45 mol C$_3$H$_6$O/400 g × 1000 g/1 kg = 8.63 m; $K_f \times m =$ 5.12°C/m × 8.63 m = 44.2°C

---

**Discuss**

Some students may think that freezing points and boiling points can be depressed or elevated without end. Explain that as the concentration of a solute, such as ethylene glycol, increases, there comes a point when the quantity of solute exceeds the quantity of solvent. Ethylene glycol becomes the solvent and water becomes the solute. The trends in colligative properties begin to reflect ethylene glycol instead of water. If the solute is a solid, such as NaCl, eventually the solution becomes saturated. But even before this saturation point, the magnitude of certain colligative properties may reach a maximum.

---

**ASSESS**

**Evaluate Understanding**

Ask, Which solution has a higher boiling point, 1 mol of Al(NO$_3$)$_3$ in 1000 g of water or 1.5 mol of KCl in 1000 g of water? Have students explain their answers. (The solution of Al(NO$_3$)$_3$ has a higher boiling point because Al(NO$_3$)$_3$ dissociates into a larger number of particles.) Why is it important to distinguish between nonvolatile and volatile compounds when discussing certain colligative properties? (Volatile solutes would quickly evaporate at higher temperatures, which would change the molal concentration of the solution.)
**Section 16.4 (continued)**

### Sample Problem 16.9

**Answers**

35. $m = 1.25 \text{ mol CaCl}_2 / 1400 \text{ g} \times 1000 \text{ g} / 1 \text{ kg} = 0.893 m \text{ CaCl}_2$; Each formula unit of CaCl$_2$ dissociates into three particles; $0.893 m \text{ CaCl}_2 \times 3 = 2.68 m$; $\Delta T_b = K_b \times m = 0.512 ^\circ \text{C} / m \times 2.68 m = 1.37 ^\circ \text{C}$; 100°C + 1.37°C = 101.37°C

36. 2.00°C / 0.512°C/m = 3.91 m; Each formula unit of NaCl dissociates into two particles; 3.91 m / 2 = 1.96 m; In 1 kg of water, 1.96 mol NaCl x 58.5 g / 1 mol = 115 g NaCl

### Practice Problems Plus

**What is the boiling point of a solution containing 96.7 g of sucrose (C$_{12}$H$_{22}$O$_{11}$) in 250.0 g water at 1 atm? (100.579°C)**

For help with significant figures, go to page R59.

**Interactive Textbook** Use it to review key concepts in Section 16.4.

**ChemASAP**

### 16.4 Section Assessment

37. **Key Concept** What are two ways of expressing the ratio of solute particles to solvent particles?

38. **Key Concept** How are freezing-point depression and boiling-point elevation related to molality?

39. How many grams of sodium bromide must be dissolved in 400.0 g of water to produce a 0.500 molal solution?

40. Calculate the mole fraction of each component in a solution of 2.50 mol ethanoic acid (CH$_3$COOH) in 10.00 mol of water.

41. What is the freezing point of a solution of 12.0 g of CCl$_4$ dissolved in 750.0 g of benzene? The freezing point of benzene is 5.48°C; $K_f$ is 5.12°C/m.

496 Chapter 16

### Calculating the Boiling Point of an Ionic Solution

**What is the boiling point of a 1.50 m NaCl solution?**

1. **Analyze** List the knowns and the unknown.

   **Knowns**
   - concentration = 1.50 m NaCl
   - $K_f$ for H$_2$O = 0.512°C/m
   - $\Delta T_b = K_b \times m$

   **Unknown**
   - boiling point = ?°C

   Each formula unit of NaCl dissociates into two particles, Na$^+$ and Cl$^-$. Based on the total number of dissociated particles, the effective molality is $2 \times 1.50 m = 3.00 m$. Calculate the boiling-point elevation and then add it to 100°C.

2. **Calculate** Solve for the unknown.

   $\Delta T_b = K_b \times m = \frac{0.512°C/m \times 3.00 m}{1.54°C} = 1.54°C$

   The boiling point of the solution is 100°C + 1.54°C = 101.54°C

3. **Evaluate** Does the result make sense?

   The boiling point increases about 0.5°C for each mole of solute particles, so the total change is reasonable. Because the boiling point of water is defined as exactly 100°C, this value does not limit the number of significant figures in the solution of the problem.

### Practice Problems

35. What is the boiling point of a solution that contains 1.25 mol CaCl$_2$ in 1400 g of water? 36. What mass of NaCl would have to be dissolved in 1.000 kg of water to raise the boiling point by 2.00°C?

### Section 16.4 Assessment

37. Molality and mole fractions are two convenient ways of expressing the ratio of solute particles to solvent particles.

38. The magnitudes of the freezing-point depression ($\Delta T_f$) and the boiling-point elevation ($\Delta T_b$) of a solution are directly proportional to the molal concentration ($m$), when the solute is molecular, not ionic.

39. 20.6 g NaBr

40. $X_{CH_3COOH} = 0.200 X_{H_2O} = 0.800$

41. 4.95°C
Small-Scale Lab

Making a Solution

Purpose
To make a solution and use carefully measured data to calculate the solution’s concentration.

Materials
- solid NaCl
- water
- 50-mL volumetric flask
- balance

Procedure
Measure the mass of a clean, dry, volumetric flask. Add enough solid NaCl to approximately fill one-tenth of the volume of the flask. Measure the mass of the flask again. Half fill the flask with water and shake it gently until all the NaCl dissolves. Fill the flask with water to the 50-mL mark and measure the mass again.

Analyze
Using your experimental data, record the answers to the following questions below your data table.

1. Percent by mass tells how many grams of solute are present in 100 g of solution.
   \[
   \text{% by mass} = \frac{\text{mass of solute}}{\text{mass of solute} + \text{solvent}} \times 100\%
   \]
   a. Calculate the mass of the solute (NaCl).
   b. Calculate the mass of the solvent (water).
   c. Calculate the percent by mass of NaCl in the solution.

2. Mole fraction tells how many moles of solute are present for every 1 mol of total solution.
   \[
   \text{Mole fraction} = \frac{\text{mol NaCl}}{\text{mol NaCl} + \text{mol H}_2\text{O}}
   \]
   a. Calculate the moles of NaCl solute.
      Molar mass NaCl = 58.5 g/mol
   b. Calculate the moles of water.
      Molar mass H\textsubscript{2}O = 18 g/mol
   c. Calculate the mole fraction of your solution.

3. Molarity (M) tells how many moles of solute are dissolved in 1 L of solution.
   \[
   M = \frac{\text{mol NaCl}}{\text{L solution}}
   \]
   a. Calculate the liters of solution. 1000 mL = 1 L
   b. Calculate the molarity of the NaCl solution.

5. Density tells how many grams of solution are present in 1 mL of solution.
   \[
   \text{Density} = \frac{\text{g solution}}{\text{mL solution}}
   \]
   Calculate the density of the solution.

You’re The Chemist
The following small-scale activities allow you to develop your own procedures and analyze the results.

1. Analyze It Use a small-scale pipet to extract a sample of your NaCl solution and deliver it to a massed empty plastic bottle. Measure the mass of the bottle and fill it with water to the 50-mL line. Measure the mass of the bottle again. Calculate the concentration of this dilute solution using the same units you used to calculate the concentration of the NaCl solution. Are the results you obtained reasonable?

2. Design It! Design and carry out an experiment to make a solution of table sugar quantitatively. Calculate the concentration of the table sugar solution using the same units you used to calculate the concentration of the NaCl solution. Is the effective molality of the table sugar solution the same as the effective molality of a sodium chloride solution of the same concentration? Recall that effective molality is the concentration value used to calculate boiling-point elevation and freezing-point depression.

Expected Outcome Students make a solution and calculate the solution’s concentration in various units.

Analyze
Sample data:
- dry bottle = 15.98 g
- flask + NaCl = 22.88 g
- flask + NaCl + water = 69.09 g

1. a. 6.90 g  b. 46.21 g  c. 13.0%
2. a. 0.118 mol NaCl  b. 2.57 mol H\textsubscript{2}O
   c. 0.0439

For Enrichment
Students could compare the freezing point depressions of the sugar and salt solutions.
(One way to do this is to partially freeze the solutions and determine the temperature of the mixtures of liquid and solid.)

2. Sample data:
   mass of dry flask = 16.72 g
   mass of flask + sugar = 20.85 g
   mass of flask + sugar + water = 69.53 g
   mass of sugar = 4.13 g
   mass of solvent = 48.68 g
   percent mass of sugar = 7.82%
   moles of sugar = 0.0121 mol
   moles of water = 2.70 mol
   mole fraction = 4.46 \times 10^{-3}
   molality = 0.249 M

molarity = 0.242 M
density = 1.1 g/mL
No, the effective molality of the NaCl solution is twice the effective molality of the sugar solution.

You’re the Chemist
1. Sample data:
   dry flask = 15.98 g
   flask + NaCl solution = 22.88 g
   flask + NaCl solution + water = 69.09 g
   mass of NaCl solution = 2.09 g
   mass of water = 47.77 g
   mass of NaCl = 0.271 g
   percent mass of NaCl = 0.544%
   moles NaCl = 4.64 \times 10^{-3} mol
   moles of water = 49.59 g
   mole fraction = 1.68 \times 10^{-3}
   molality = 0.0936 M
   density = 1.0 g/mL